

SUBJECT MATTER

The study of the linear equation systems consists of four sections.

In the first section, we define the *matrices* and study their properties. Preliminary, we introduce the *vector* (or *linear*) *space* X over a field K , by providing X with a suitable binary operation, called *addition*, and with a suitable operation called *scalar multiplication*. Such operations must obey two *distributive laws*. The elements of X are called *vectors* (or *points*), while the elements of K are called *scalars*. If $K = \mathbb{R}$, X is called *real vector* (or *linear*) *space*. A very important example of vector space over a field is the *vector space* \mathbb{R}^n over the real field. For the vector space \mathbb{R}^n over the real field we define:

- the *linear combination* of vectors $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_k \in \mathbb{R}^n$ according coefficients $c_1, c_2, \dots, c_k \in \mathbb{R}$
- vectors $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_k \in \mathbb{R}^n$ *linearly independent*
- vectors $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_k \in \mathbb{R}^n$ *linearly dependent*

and underline their properties. Moreover, we define the *dimension* and the *basis* of such real vector space.

After that, in the first section we define and underline the properties of

- the *rectangular real matrix* (of dimension $m \times n$)
- the *square real matrix of order* n
- some particular matrix (*column matrix, row matrix, diagonal matrix, unit matrix, upper triangular matrix, lower triangular matrix, transpose matrix, symmetric matrix, skew-symmetric matrix*)
- the *sum* (or *addition*) of two matrices
- the *product* (or *multiplication*) of a matrix by a real number
- the *product* (or *multiplication*) of two rectangular matrices
- the *invertible* matrix and the *inverse* matrix

In the second section, we define the *determinants* and study their properties. Preliminary, we introduce the *combinatorial analysis*. Let $n \in \mathbb{N}$, $k \in \{1, \dots, n\}$, A be the set constituted by n distinct elements a_1, a_2, \dots, a_n . We define and underline the properties of

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- the *k*-combination of the set *A*, the *k*-disposition of the set *A*, the permutation of the set *A*
- the *inversion* (both *even* and *odd*) of a permutation.

After that, in the second section we define and underline (also with examples) the properties of

- the *determinant* of order *n*, the determinant of a *singular* matrix, the determinant of a *non-singular* matrix, the rule of *Sarrus*, the *minor* of a determinant of order *n*, the *complementary minor*, the *algebraic complement* of a minor determinant, the *rank* of a rectangular matrix, the *rank* of the transpose matrix, the *adjoint matrix* of a square matrix.

Moreover, we study

- the determinant of the transpose matrix, the sufficient conditions to annul a determinant, the determinant of the matrix sum, the determinant of the matrix product, the *Rule of Laplace* to calculate a determinant, the *linear dependence* (or *independence*) of the *rows* (or *columns*) of a square matrix.

In the third section, we define and study the properties of

- the *linear equation systems*, the *matrix of coefficients* of the system, the *column of the known terms*, the *column of the unknowns*, the *augmented matrix*, the *consistent* (or *inconsistent*) linear equation system, the *homogeneous* linear equation system.

Moreover, we demonstrate, to solve a linear equation system

- the *rule of Cramer*, the theorem of *Rouché – Capelli*, the *elimination procedure of Gauss*.

In the fourth section, we present another important problem of the matrix theory: the *eigenvalue problem*. Such problem occurs in various problems of *Geometry*, *Mechanics*, *Astronomy*, *Physics*. We define and study the properties of the *eigenvalues* of a square matrix, the *characteristic equation* (also called *secular equation*) of a square matrix, the *characteristic polynomial* of a square matrix, the *eigenvectors* of a square matrix.